**IGCSE Further Maths – Matrices**

**Exercise 1 – Matrix Multiplication (questions from AQA Worksheets)**

**Question 1**

Work out

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **(a)** |  | **(b)** |  | **(c)** | 2 |
| **(d)** |  | **(e)** | 6 | **(f)** |  |

**Question 2**

 **A** = **B** =  **C** = 

Work out

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **(a)** | **AB** | **(b)** | **BC** | **(c)** | 3**A** |
| **(d)** | **BA** | **(e)** | −**C** | **(f)** | **B** |

**Question 3**

 **P** = **Q** =  **C** = 

Work out

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **(a)** | **P** 2 | **(b)** | **QP** | **(c)** | 5**Q** |
| **(d)** | **PC** | **(e)** | **IQ** | **(f)** | 3**I** |

**Question 4**

Work out

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **(a)** |  | **(b)** |  | **(c)** |  |
| **(d)** |  | **(e)** |  | **(f)** |  |

**Question 5 (Non-calculator)**

Work out, giving your answers as simply as possible.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **(a)** |  | **(b)** |  | **(c)** |  |
| **(d)** |  | **(e)** |  | **(f)** |  |

**Question 6**

Work out, giving your answers as simply as possible.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **(a)** |  | **(b)** |  | **(c)** |  |
| **(d)** |  | **(e)** |  | **(f)** |  |

**Question 7**

Work out, giving your answers as simply as possible.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **(a)** |  | **(b)** |  | **(c)** |  |
| **(d)** |  | **(e)** |  | **(f)** |  |

**Exercise 1b**

**Question 1**



**Question 2**

1. Work out $\left(\begin{matrix}2&-1\\\frac{1}{3}&0\end{matrix}\right)\left(\begin{matrix}0&b\\a&c\end{matrix}\right)$ **Give your answer in terms of** $a,b,c$**.**
2. **If** $\left(\begin{matrix}2&-1\\\frac{1}{3}&0\end{matrix}\right)\left(\begin{matrix}0&b\\a&c\end{matrix}\right)=I$ where $I$ is the identity matrix, work out the values of $a,b,c$.

**Question 3**



**Question 4**



**Exercise 2**

1. [Jan 2013 Paper 2 Q15] Describe fully the **single** transformation represented by the matrix $\left(\begin{matrix}0&-1\\1&0\end{matrix}\right)$
2. [Set 2 Paper 1 Q4] The transformation matrix $\left(\begin{matrix}a&2\\-1&1\end{matrix}\right)$ maps the point $\left(3,4\right)$ onto the point $\left(2,b\right)$. Work out the values of $a$ and $b$.
3. [Set 3 Paper 1 Q6] The matrix $\left(\begin{matrix}a&b\\-a&2b\end{matrix}\right)$ maps the point $\left(5,4\right)$ onto the point $\left(1,17\right)$. Work out the values of $a$ and $b$.
4. [Worksheet 2 Q5] Work out the image of the point *D* (−1, 2) after transformation by the matrix $\left(\begin{matrix}2&3\\-1&1\end{matrix}\right)$
5. [Worksheet 2 Q6] The point *A*(*m*, *n*) is transformed to the point *A*′ (−2, 0) by the matrix $\left(\begin{matrix}2&3\\1&1\end{matrix}\right)$
Work out the values of *m* and *n*.
6. [Worksheet 2 Q8] Describe fully the transformation given by the matrix $\left(\begin{matrix}0&-1\\-1&0\end{matrix}\right)$
7. [Worksheet 2 Q9] The unit square *OABC* is transformed by the matrix $\left(\begin{matrix}h&0\\0&h\end{matrix}\right)$ to the square *OA*′*B*′*C*′.
The area of *OA*′*B*′*C*′ is 27. Work out the exact value of *h*.
8. [Specimen Paper 2 Q20] (a) Matrix $A=\left(\begin{matrix}4&3\\1&1\end{matrix}\right)$
Work out the image of point $P\left(2,-1\right)$ using transformation matrix $A$.
(b) Point $Q$ is $\left(0,1\right)$
 Line $PQ$ is transformed to line $P'Q'$ using matrix $A$.
 
 Work out the length of $P'Q'$.

**Exercise 3**

1. Point $\left(3,-2\right)$ is transformed by the matrix $\left(\begin{matrix}1&-1\\0&1\end{matrix}\right)$ followed by a further transformation by the matrix $\left(\begin{matrix}0&2\\1&0\end{matrix}\right)$.
(i) Work out the matrix for the combined transformation.
(ii) Work out the co-ordinates of the image point of $P$.
2. Point $\left(-1,4\right)$ is transformed by the matrix $\left(\begin{matrix}3&-1\\-2&2\end{matrix}\right)$ followed by a further transformation by the matrix $\left(\begin{matrix}1&0\\3&-2\end{matrix}\right)$.
(i) Work out the matrix for the combined transformation.
(ii) Work out the co-ordinates of the image point of $W$.
3. The unit square is reflected in the $x$-axis followed by a rotation through $180°$ centre the origin. Work out the matrix for the combined transformation.
4. The unit square is enlarged, centre the origin, scale factor 2 followed by a reflection in the line $y=x$. Work out the matrix for the combined transformation.
5. [Jan 2013 Paper 2 Q17] $\left(\begin{matrix}-1&0\\0&1\end{matrix}\right)$ represents a reflection in the $y$-axis. $\left(\begin{matrix}0&1\\1&0\end{matrix}\right)$ represents a reflection in the line $y=x$.
Work out the matrix that represents a reflection in the $y$-axis followed by a reflection in the line $y=x$.
6. [June 2012 Paper Q22] The transformation matrix $\left(\begin{matrix}0&-1\\-1&0\end{matrix}\right)$ maps a point $P$ to $Q$. The transformation matrix $\left(\begin{matrix}1&0\\0&-1\end{matrix}\right)$ maps point $Q$ to point $R$.
Point $R$ is $\left(-4,3\right)$. Work out the coordinates of point $P$.
7. [Set 1 Paper Q14b] The unit square OABC is transformed by reflection in the line $y=x$ followed by enlargement about the origin with scale factor 2. What is the matrix of the combined transformation?
8. $A=\left(\begin{matrix}3&0\\0&3\end{matrix}\right)$ and $B=\left(\begin{matrix}-1&0\\0&1\end{matrix}\right)$.
The point $P\left(2,7\right)$ is transformed by matrix $BA$ to $P'$. Show that $P'$ lies on the line
$7x+2y=0$.