

## Using Venn Diagrams to determine the LCM/HCF

“Determine the LCM and HCF of 60 and 72.”

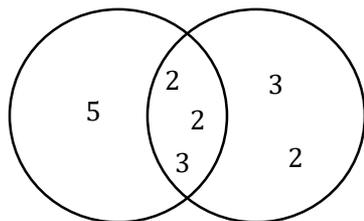
### The Venn Diagram Method

- 1 First find the prime factorisation of each number:

$$60 = 2^2 \times 3 \times 5$$

$$72 = 2^3 \times 3^2$$

- 2 Form a Venn diagram:



- 3 Use intersection for HCF:

$$2^2 \times 3 = 12$$

And union for LCM:

$$2^3 \times 3^2 \times 5 = 360$$

### Why I don't like it...

- I have no 'mathematical' objection to this method, and the Venn Diagram nicely conveys the idea that the intersection, i.e. 'shared' factors forms the highest 'common' factor. My misgivings are therefore more practical...
- My experience is that students ALWAYS forget the method.
- It is incredibly easy to make an error.
- Forming the Venn Diagram is time-consuming and can be difficult (not to mention time-consuming to teach).

### Preferred Method: "What Wins, What Loses"

As before, students find the prime factorisation of each number, but this time, ensure the same prime factors are vertically aligned, i.e. the 2s appear in the same column, etc.

For each prime factor we see what 'loses' to determine the HCF (more formally, what goes into both), where 'nothing' loses against something:  $2^2$  'loses' against  $2^3$ , 3 loses against  $3^2$  and nothing loses against 5

$$\therefore HCF = 2^2 \times 3 = 12$$

Similarly we see what 'wins' (formally: what both go into) to determine the LCM:

$2^3$  beats  $2^2$ ,  $3^2$  beats 3 and 5 beats nothing:

$$\therefore LCM = 2^3 \times 3^2 \times 5 = 360$$

### Why I think it works better:

- Method much easier and retention better.
- Nicely conveys idea that  $a^b$  is a factor of  $a^c$  provided  $b \leq c$ . Students experience this in factorisation, e.g. with  $x^2 + x^3$ , they have a common factor of  $x^2$ . So "what loses out of  $2^2$  and  $2^3$ " is effectively the same as saying "what is common to  $2^2$  and  $2^3$ ". The 'wins/loses' just gives the method a snappy name which makes it easier to remember.