

GCSE Sets

Exercise 1 – Constructing Venn Diagrams

1. $\xi = \{4,5,6,7,8\}$,
 $A = \{5,6,7\}$
 $B = \{6,7,8\}$

Construct a Venn Diagram to show these sets.

2. $\xi = \{1,2,3,4,5,6,7,8\}$
 $A = \{1,2,3,8\}$,
 $B = \{3,4,5,8\}$
 $C = \{1,5,6,8\}$

Construct a Venn Diagram to show these sets.

3. $\xi = \{1,2, \dots, 10\}$
 $A = \text{set of all primes}$
 $B = \text{triangular numbers}$
 $C = 1 \text{ less than multiple of } 4$
Construct a Venn diagram for these sets.

4. You have two sets A and B and $B \subset A$. Draw a Venn Diagram (without any numbers) that indicates the relationship between the sets.
5. You have three sets A , B and C and $A \subset C$. Draw a Venn Diagram (without any numbers) that indicates the relationship between the sets.

* The **power set** of a set is the set of all possible subsets, including the empty set and itself. E.g.

$$P(\{1,2\}) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$$

- a. Determine $P(\{1,2,3\})$
b. Determine how many members $P(A)$ has for a set of A of size n .

Exercise 2 – Venn Diagrams involving Frequencies

Test Your Understanding question:

[Jan 2012 Q6] The following shows the results of a survey on the types of exercise taken by a group of 100 people.

65 run 48 swim 60 cycle
40 run and swim 30 swim and cycle
35 run and cycle 25 do all three

- (a) Draw a Venn Diagram to represent these data. (4)

Find the probability that a randomly selected person from the survey

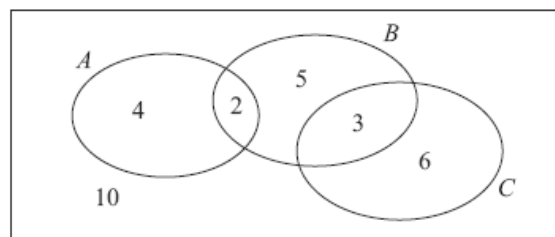
- (b) takes none of these types of exercise, (2)
(c) swims but does not run, (2)
(d) takes at least two of these types of exercise. (2)

Jason is one of the above group.

Given that Jason runs,

- (e) find the probability that he swims but does not cycle. (3)

1. [JMC 2002 Q11] The Pythagoras School of Music has 100 students. Of these, 60 are in the band and 20 are in the orchestra. Given that 12 students are in both the band and the orchestra, how many are in neither the band nor the orchestra?
2. [Edexcel S1 May 2010 Q4] The Venn diagram in Figure 1 shows the number of students in a class who read any of 3 popular magazines A , B and C .



One of these students is selected at random.

- (a) Show that the probability that the student reads more than one magazine is $\frac{1}{6}$. (2)
(b) Find the probability that the student reads A or B (or both). (2)
(c) Write down the probability that the student reads both A and C . (1)

Given that the student reads at least one of the magazines,

- (d) find the probability that the student reads C . (2)

3. [Edexcel S1 Jan 2010 Q4] There are 180 students at a college following a general course in computing. Students on this course can choose to take up to three extra options.

112 take systems support,
70 take developing software,
81 take networking,
35 take developing software and systems support,
28 take networking and developing software,
40 take systems support and networking,
4 take all three extra options.

- (a) Draw a Venn diagram to represent this information. (5)

A student from the course is chosen at random. Find the probability that the student takes

- (b) none of the three extra options, (1)
(c) networking only. (1)

Students who want to become technicians take systems support and networking. Given that a randomly chosen student wants to become a technician,

- (d) find the probability that this student takes all three extra options. (2)

4. [Edexcel S1 May 2008 Q5] A person's blood group is determined by whether or not it contains any of 3 substances A, B and C.

A doctor surveyed 300 patients' blood and produced the table below.

Blood contains	No. of Patients
only C	100
A and C but not B	100
only A	30
B and C but not A	25
only B	12
A, B and C	10
A and B but not C	3

- (a) Draw a Venn diagram to represent this information. (4)

- (b) Find the probability that a randomly chosen patient's blood contains substance C. (2)

Harry is one of the patients. Given that his blood contains substance A,

- (c) find the probability that his blood contains all 3 substances. (2)

Patients whose blood contains none of these substances are called universal blood donors.

- (d) Find the probability that a randomly chosen patient is a universal blood donor. (2)

5. [Edexcel S1 Jan 2008 Q5] The following shows the results of a wine tasting survey of 100 people.

96 like wine A, 93 like wine B,
96 like wine C, 92 like A and B,
91 like B and C, 93 like A and C,
90 like all three wines.

- (a) Draw a Venn Diagram to represent these data. (6)

Find the probability that a randomly selected person from the survey likes

- (b) none of the three wines, (1)
(c) wine A but not wine B, (2)
(d) any wine in the survey except wine C, (2)
(e) exactly two of the three kinds of wine. (2)

Given that a person from the survey likes wine A,

- (f) find the probability that the person likes wine C. (3)

6. [Edexcel S1 May 2006 Q6] A group of 100 people produced the following information relating to three attributes. The attributes were wearing glasses, being left-handed and having dark hair.

Glasses were worn by 36 people, 28 were left-handed and 36 had dark hair. There were 17 who wore glasses and were left-handed, 19 who wore glasses and had dark hair and 15 who were left-handed and had dark hair. Only 10 people wore glasses, were left-handed and had dark hair.

- (a) Represent this on a Venn diagram. (6)

A person was selected at random from this group. Find the probability that this person

- (b) wore glasses but was not left-handed and did not have dark hair, (1)
(c) did not wear glasses, was not left-handed and did not have dark hair, (1)
(d) had only two of the attributes, (2)
(e) wore glasses, given they were left-handed and had dark hair. (3)

7. [Edexcel S1 Jan 2005 Q5] Articles made on a lathe are subject to three kinds of defect, A, B or C. A sample of 1000 articles was inspected and the following results were obtained.

31 had a type A defect
37 had a type B defect
42 had a type C defect
11 had both type A and type B defects
13 had both type B and type C defects
10 had both type A and type C defects
6 had all three types of defect.

- (a) Draw a Venn diagram to represent this. (6)

Find the probability that a randomly selected article from this sample had

- (b) no defects, (1)
(c) no more than one of these defects. (2)

An article selected at random from this sample had only one defect.

- (d) Find the probability that it was a type B defect. (2)

Two different articles were selected at random from this sample.

- (e) Find the probability that both had type B defects. (2)

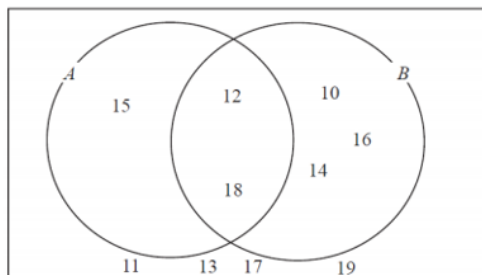
8. [SMC 2011 Q17] Jamie conducted a survey on the food preferences of pupils at a school and discovered that 70% of the pupils like pears, 75% like oranges, 80% like bananas and 85% like apples. What is the smallest possible percentage of pupils who like all four of these fruits?

- A. At least 10% B. At least 15%
C. At least 20% D. At least 25%
E. At least 70%

Exercise 3 – Combining Sets

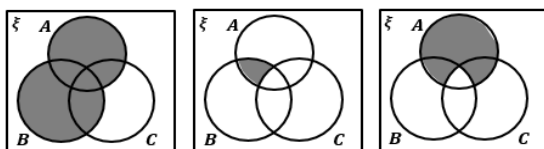
Test Your Understanding

1. [Edexcel] Here is a Venn diagram.

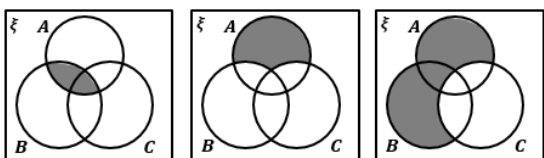


- Write down the numbers that are in the set $A \cup B$.
 - One of the numbers in the diagram is chosen at random. Find the probability that the number is in set A' .
2. $\xi = \{ \text{all whole numbers} \}$
 $A = \{ \text{factors of 60} \}$ $B = \{ \text{multiples of 3} \}$
 List the members of the set $A \cap B$.

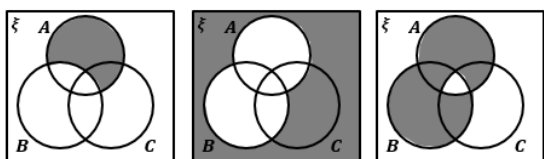
Matching Activity



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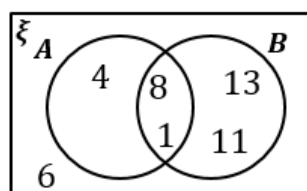
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Choose from the following for each region.

- $A \cap B'$
- $A \cap B' \cap C'$
- $A \cup B$
- $A' \cap B'$
- $A \cap B$
- $A \cap (B \cap C)'$
- $(A \cup B) \cap (A \cap B \cap C)'$
- $A \cap B \cap C'$
- $(A \cup B) \cap C'$

Main Exercise

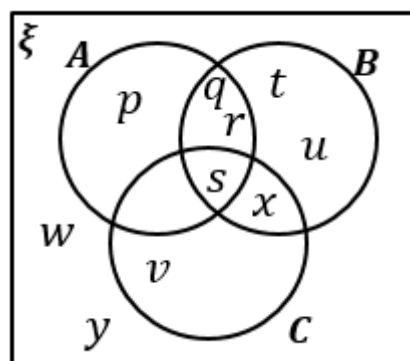
1.



List the numbers in:

- $A \cap B$
- $A \cup B$
- A'
- $A' \cap B$
- Given that a number is chosen at random, find the probability it is in set B .

2.



Determine the sets:

- $A \cap B$
 - $A \cup C$
 - $A' \cap C'$
 - $A \cap B \cap C$
 - $A \cap B' \cap C$
 - $(A \cup C) \cap B'$
 - Given that a number is chosen at random from the set $A \cup C$, find the probability it is in set B .
3. A and B are two sets such that:
- $$n(\xi) = 20$$
- $$n(A) = 14$$
- $$n(A \cap B) = 3$$
- $$n(B') = 12$$

Form a Venn diagram, where the number in each region is the **number** of elements in it.

4. $A = \{f, r, o, s, t\}$
 $B = \{b, a, r, t, o, n\}$

Determine the sets:

- $A \cap B$
- $A \cup B$
- $A' \cap B$

5. $\xi = \{1,2,3,4,5,6,7,8,9,10\}$

$A = \{ \text{all prime numbers} \}$

$B = \{ \text{all multiples of 6} \}$

Determine:

a. $A \cup B$

b. $A \cap B$

6. $\xi = \{1,2,3,4,5,6,7,8,9,10\}$

$A = \{ \text{all even numbers} \}$

$B = \{ \text{all factors of 8} \}$

Determine:

a. $A \cup B$

b. $A \cap B$

c. $A \cap B'$

7. Construct a Venn Diagram for sets

A, B, C (without numbers) such that:

a. $A \subset B, B \cap C = \emptyset$

b. $A \subset B, A \cap C = \emptyset$

✂ A, B, C are sets such that:

$$n(A) = 20,$$

$$n(B) = 20,$$

$$n(C) = 20,$$

$$n(A \cap B) = 10$$

$$n(A \cap C) = 10$$

$$n(B \cap C) = 10$$

$$n(A \cap B \cap C) = 5$$

Determine $n(A \cup B \cup C)$.